Position paper on Mathematics Education and Computational thinking

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1. Introduction

Mathematics has become intrinsically intertwined with our lives. "*The mathematization of modern society and modern life has been growing exponentially, so that by now virtually the whole range of human activities and institutions are conceptualised and regulated numerically, including sport, popular media, health, education, government, politics, business, commercial production, and science*." (Ernest, 2000).

NEP 2020 document in its introductory note has highlighted that, "The need for a skilled workforce, particularly involving mathematics, computer science, and data science, in conjunction with multidisciplinary abilities across the sciences, social sciences, and humanities, will be increasingly in greater demand."

1.1Position of teaching mathematics at school level

It thus becomes even more imperative that we not only improve the quality but also make the necessary adaptations to school mathematics to prepare students for life in the 21st Century.

To realise the vision of NEP 2020 – which encapsulates both the country's educational aims and also the Sustainable Development Goals – we envisage a school mathematics that equips learners with a holistic mathematical literacy that enables them to

(i) successfully navigate their way through life;
(ii) participate fully in an increasingly interconnected and technological world and to read the world through mathematics for promoting social justice and equity and
(iii) carry forward our traditions while defining worthwhile values and creating a new culture.

To achieve this, priority will be towards achieving foundational numeracy by 2025-26. The curriculum will require appropriate streamlining at all the stages of schooling. Pedagogical processes need to be as experiential as possible, with more emphasis on computational thinking as it not only develops the essential mindset for problem stating and problem solving in mathematics but also develops transversal 21st Century skills that are necessary across subjects and for the development of the individual.

Learning mathematics is very essential for life. To start with, at the formative stage, quantitative or numerical literacy is a must and based on the interest of the learner and their competency, one can progress to advance levels. However, the fact is that it is very essential to provide mathematics education to all.

This challenge is all the truer for 21st century, as the emphasis is on Science/Technology/ Engineering/ Mathematics (S-T-E-M). Mathematics being the foundation of STEM as well as STEAM, and innovation, the situation requires urgent attention.

Why teach mathematics for 21st century?

- Most jobs involve mathematics.
- Most other subjects use mathematics.
- Higher education requires mathematics.
- Science, technology, and engineering depends on mathematics.
- Humanities are integrated with mathematics.
- Real life problems are solved based on mathematics.
- Mathematics helps to think and reason clearly and logically.
- Business and commercial transactions require mathematics.

Hence, everybody needs mathematics everywhere for 21st century. In the 21st century, "it is no longer about what and how much a student knows about mathematics, it's about what they can do with what they know about mathematics".

1.2 Nature of Mathematics

"Mathematics, as an expression of the human mind, reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality"- Courant and Robbins.

Mathematical objects and ideas are abstract - created by humans from the needs of science and daily life. They have no physical properties like shape, size, colour, sound and so on, which leads to the problem of making them experiential. Mathematical ideas are formed by classifying similarly related and commonly noticed properties. Number, a root concept, is derived by providing experiences which would represent the property that all collections of that number of objects have in common. The concept of addition is built on the concept of number, and it then becomes the pre-requisite concept for viewing multiplication as repeated addition. This in turn builds on to understanding higher concepts. We see clearly that the concept network in mathematics is hierarchical and cumulative.

To communicate mathematical ideas, mathematicians have, over the years, developed the language of mathematics which has vocabulary, symbols, and sentence structure. This helps in the mathematical communication of ideas or concepts during discussions in the classroom and leads to representing ideas, observing, and generalizing patterns, communicating thought processes, and justifying their discoveries and learning in age-appropriate mathematical language.

1.3. Computational Thinking

Computational thinking is all about the thought process involved in problem posing and problem solving. *It is an analytical thinking skill that draws on concepts from computer science but is a fundamental skill useful for everyone* (Wing [2006]).

In fact, it can be taught without a device, and it can, and should, be taught right from the foundational stage when it is integrated seamlessly with the regular routines and curriculum.

Computational thinking involves (i) **Decomposition**: breaking down a complex problem into smaller, simpler problems, (ii) **Pattern recognition:** looking for similarities among and within problems and making connections to their experiences, (iii) **Abstraction**: focusing on important facts and ignoring unrelated details and (iv) Design and use of **algorithms**: designing and using simple problem-solving steps.
The integration of computational thinking in the classroom is therefore simply a conscious awareness of developing these skills in the learner. Whether it is in the English, Science or Mathematics classroom, whether the child is learning a poem, the periodic table or multiplication tables, the four components of computational thinking may be integrated into the process so that the learner is able to gain conceptual clarity and at the same time devise and use procedures rather than simply memorise them.

There are many ways in which we can integrate and encourage computational thinking in the classroom. At the Foundational Stage, children can act out or describe the instructions of any familiar activity, such as putting on shoes, eating food, etc. At the Preparatory Stage, children can describe their favourite toy, activity, game, place, person, etc., which will help them think about important attributes that will clearly describe the thing. Students can formulate their own questions and make algorithms to analyse data at the Middle Stage. Secondary Stage students can write an algorithm (or a precise sequence of steps) on how to do matrix multiplication or how to solve a quadratic equation.
Computational thinking will thrive in a teaching atmosphere which is open, safe, and flexible and encourages group work, communication, investigation, and exploration.

NEP 2020 (4.25) emphasises the significance of mathematical and computational thinking as, “It is recognized that mathematics and mathematical thinking will be very important for India’s future and India’s leadership role in the numerous upcoming fields and professions that will involve artificial intelligence, machine

learning, and data science, etc. Thus, mathematics and computational thinking will be given increased emphasis throughout the school years, starting with the foundational stage, through a variety of innovative methods, including the regular use of puzzles and games that make mathematical thinking more enjoyable and engaging. Activities involving coding will be introduced in Middle Stage."

1.4 Process Skills

The focus of our current classroom practices and pedagogy is on content and the teaching of mathematical procedures and neglects the development of mathematical processes such as formal problem solving, use of heuristics, estimation and approximation, optimization, use of patterns, visualisation, representation, reasoning, and proof, making connections and mathematical communication, which have also been mentioned in the NCF 2005.

It is important to have a mathematics curriculum that is aligned to the higher aim of developing critical thinkers and problem solvers. A focus on mathematical processes will bring a shift from practices such as rote learning, memorization of formulas and the regurgitation of class work to meaning making, doing important mathematics, and learning to mathematize.

1.5 Problem Solving: This is an important life skill which the school should develop across the four stages of schooling. The learner should construct mathematical knowledge by solving problems that he or she comes across in other contexts. The components of computational thinking will systematise the teaching and application of problem-solving skills.

Students must be trained to work independently to explore and discover. Heuristics is the heart of problem solving and it encourages independent thinking, investigating and enquiry to build knowledge thus making the student responsible for his/her learning. This method helps to develop the students' problems solving attitude and cognitive abilities that ultimately pushes them to think mathematically.

1.6 Reasoning and Proof: Reasoning is foundational in constructing mathematical knowledge. Asking *Why* is key to this. Encouraging reasoning at all stages is a way to get students to think and respond. Reasoning could be abstract or

quantitative. One could see it as constructing valid arguments or critiquing the reasoning of others, including the reasoning behind mathematical procedures.

Proof is an important part of mathematics. School mathematics focuses on deductive proofs. Rigor in proof begins at the secondary stage, but it is important that students at every stage are encouraged to develop from reasoning to proof.

1.7 Visualisation and Representation: Mathematical ideas are abstract and may be modelled using materials, pictures, symbols, written/spoken language, real-life situations, tables, graphs, etc. Such visualisation and representation support in arriving at a solution by moving step by step through reasoning. This is fundamental and one uses such representation based on how one understands the problem in the context. This also caters to learners with different learning preferences. Students at all stages should be introduced and encouraged to present ideas through representations. Eventually, they make use of this process skill to interpret physical, social, and mathematical problems.

1.8 Connections: Learning is always associated with making connections. Connections could be between prior to current knowledge, between mathematical concepts, between mathematics and other subjects, and between mathematics and real-life context. Such connections make learning meaningful and understanding deeper that in turn helps student view the mathematical structures in whole. At every stage, such connections need to be demonstrated in classroom.

1.9 Communication: Mathematical communication is the use of the language of mathematics to share mathematical ideas, clarify understanding and ask questions. This kind of communication gets students to reflect, refine, discuss, and modify ideas thus increasing understanding and developing the stamina to explore in the subject. This is seen in a classroom that encourages a culture of respectful mathematical argument and discussion - a space where all views are accepted without threat and facilitates the learner's struggles to articulate and solve.

1.10 Approximation, estimation, and optimisation: Estimation and approximation are skills practised unconsciously by almost everyone daily.

Estimation is the mental skill to make an educated guess regarding quantity or measure while approximation is about providing a roughly correct answer.
Mathematical optimization is the use of strategies and techniques to evaluate a problem or a context and make an educated decision. This skill can be developed in school at all stages by encouraging students to come up with the best solution with the information available.

2. Vision and Goals of school mathematics education

In defining a vision for school mathematics, with reference to the NEP 2020, four strands or themes are identified. These are **Appreciation**, **Development**, **Connection** and **Inclusion** and they are intertwined with the Nature of Mathematics.

Appreciation: School mathematics education should enable an appreciation and understanding of what mathematics is. The underlying implication is that education functionaries, teacher educators, teachers, indeed anyone connected with the field of school mathematics education should understand the nature of mathematics- characterized by the abstract and hierarchical nature of mathematical concepts, symbolic language, logical reasoning, and proof. This understanding will flow into every aspect of the learner's experience of mathematics – be it in pedagogy or pedagogical materials.
Understanding and harnessing the nature of mathematics will lead to a perception that mathematics is aesthetically pleasing and that mathematical truths are based on rigorous and well-defended reasoning. At the same time, space is provided for constructing new mathematical knowledge. Unfortunately, at present, mathematics is not a popular subject for it has been plagued by the perception that it is difficult, and learners approach it with inherited prejudices. Such an unproductive and harmful perception must give way to a robust appreciation of what mathematics is and how the study of mathematics can change lives and societies.

Development: The study of mathematics would be incomplete without the development of the individual learner and of society. A mindful approach to the pedagogy of mathematics can develop ethical values and an understanding of one's culture, community and ethos leading to the inculcation of qualities of good citizenship in the learner. A sound knowledge of mathematics enables an

individual to handle data analytically, understand trends and take informed decisions which could be of benefit to the nation and to the world. School mathematics education lays the foundation for this and an encouraging and enabling classroom will allow children to investigate boldly, analyse and think critically, work creatively, and communicate logically. Further, the professional aspirations of the learner must be enabled by the study of the subject. Though mathematics is popularly seen as a stepping-stone to professional courses in related fields, higher studies in mathematics should be perceived as a viable option leading to good career choices.

Connection: An education that focuses on making connections rather than ensuring compliance has the potential to make a difference and the theme of connections is an important strand in the landscape of school mathematics education. The incorporation of our country's rich culture, traditions, and history as well as indigenous knowledge in the curriculum enables the learner to connect mathematics with his or her own societal and family practices. When the learner is placed at the centre of the learning and has intentional engagement with pedagogy that is related to his or her context, there is scope to surface and build on an intuitive understanding of mathematics. Such an approach will result in "*children's development of the ability to engage in sense-making in and with mathematics, a deeper understanding of mathematical ideas, the ability to use mathematical ideas productively in solving problems, and a more positive view both of mathematics and of themselves as sense-makers in mathematics*." (Schoenfeld, 2011).
Mathematics, like every other subject is not an island and connections from it to other subjects which are obviously (such as science and economics) and subtly (such as geography and political science) connected to it must reflect in discussions, explorations and even such simple activities as reading a post on social media. Vocational courses, sport and art activities can be opportunities for mathematization – to recognize and investigate patterns, to identify constants and variables, to reason and deduce and predict. Here the role of computational thinking with its four strands of decomposing, pattern recognition, abstraction and algorithms may be harnessed to maximise learning opportunities.

Inclusion: The current perception of mathematics is that it can only be mastered by a select few. An inclusive approach would make it equitable and accessible to

all. Barriers need to come down across geographies – so that learners in remote regions of the country have access to contextual, accessible material, across different languages and dialects, across gender so that girls continue to engage with a subject considered to be an exclusive male preserve, across abilities so that every learner can benefit from his or her study of the subject and across differences in economic backgrounds as well as the digital divide.

2.1Goals of teaching mathematics

The main goal of teaching mathematics at school level is the mathematization of child's thinking and thought processes. The kind of thinking that child learns in mathematics is an ability to handle abstractions and an approach to problem solving. According to George Polyo, the two major goals of school mathematics education are:

- **The narrow goal,** relating to numeracy. This is to develop useful capabilities related to numbers, number operations, measurement, decimals, percentages etc. Narrow or Primary goals are that of turning out employable adults who eventually contribute to social and economic development.
- **The higher goal,** that of developing the inner resources of the child. It is the role that mathematics plays in developing thinking and problem- solving skills. The higher goal is to develop the child's resources to think and reason mathematically, to pursue assumptions to logical conclusions and to handle abstraction.

The role of mathematics teaching in the curricular framework for 21st century education is positioned on these twin goals: narrow goal and higher goal.

That is, what mathematics education can do to engage the mind of every student, and how it can strengthen the student's inner resources. We also describe the vision of mathematics education at school level on the same twin perspectives.

A culture of awareness of mathematics – its pervasiveness, aesthetic value, nature, and usefulness must evolve. Our daily activities, academic subjects and vocational courses are entwined with mathematical concepts, but these remain unharnessed unless there is an explicit focus on it.

i) Change in the perception of mathematics – from an exclusive to an inclusive domain ensuring that all citizens are mathematically literate.

ii) Mathematics education which caters to differentiated learning, variety in pedagogical strategies and the development of a culture of rational discussion, mindful use of data and logical reasoning.
iii) A robust assessment system that tracks progress in mathematics and identifies and addresses causes of concern.
iv) A curriculum which incorporates the culture and traditions as well as indigenous knowledge in related curricular material and pedagogical strategies.

3. Background of mathematics teaching and learning:

The Kothari commission way back stressed on the need for mathematics and science in school as well as in higher education and emphasized the importance of children learning mathematics for the development of science and technology and for industrial growth.

The 1968 and 1986 National Policies of Education spoke in the same tone as the Kothari Commission report and the 1986 policy states that "Mathematics should be visualized as the vehicle to train a child to think, reason, analyze and articulate logically. Apart from being a specific subject, it should be treated as a concomitant to any subject involving analysis and reasoning" (Government of India– Ministry of Human Resource Development, 1998, p.29).

National Curriculum Framework 2005 states that Mathematics is an 'exact science'. The estimation of quantities and approximating solutions is also an essential skill. Visualization and representation are skills that Mathematics can help to develop. Modelling situations using quantities, shapes and forms are the best use of Mathematics. mathematical concepts can be represented in multiple ways, and these representations can serve a variety of purposes in different contexts.

According to the National Curriculum Framework (NCF) 2005, the main goal of Mathematics education in schools is the 'Mathematisation' of a child's thinking. Clarity of thought and pursuing assumptions to logical conclusions is central to the mathematical enterprise.

The NCF also speaks of mathematical communication – that it is precise and employs unambiguous use of language and rigor in formulation, which are important characteristics of mathematical treatment.

The aims of teaching and learning mathematics are to encourage and enable students to: recognize that mathematics permeates the world around us. appreciate the usefulness, power and beauty of mathematics. enjoy mathematics and develop patience and persistence when solving problems.

NCF focuses on mathematics education as follows:

- seek narrow aims of teaching Mathematics in school.
- seek higher aims of teaching Mathematics in school.
- teach more than one way of solving problems of Mathematics.
- teach variety of Mathematics like arithmetic, algebra, geometry and data handling.

The NCF envisions school Mathematics as taking place in a situation where:

1. Children learn to enjoy Mathematics rather than fear it.

2. Children learn "important" Mathematics which is more than formulas and mechanical procedures.

3. Children see Mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.

4. Children pose and solve meaningful problems.

5. Children use abstractions to perceive relationships, to see structures, to reason out things, to argue the truth or falsity of statements.

6. Children understand the basic structure of Mathematics: arithmetic, algebra, geometry and trigonometry, the basic content areas of school Mathematics, all of which offer a methodology for abstraction, structuration and generalization.

7. Teachers are expected to engage every child in class with the conviction that everyone can learn Mathematics

Problems in teaching and learning of mathematics

1. A sense of fear and failure regarding Mathematics among a majority of children.

2. A curriculum that disappoints both a talented minority as well as the non-participating majority at the same time.

3. Crude methods of assessment and evaluation that encourage the perception of Mathematics has mechanical computation - problems, exercises, methods of evaluation are mechanical and repetitive with too much emphasis on computation.

4. Lack of teacher preparation and support in the teaching of Mathematics.

5. Structures of social discrimination that get reflected in Mathematics education often leading to stereotypes like 'boys are better at Mathematics than girls.

6. lack explicit the connections between mathematics and other subjects.

7. Disappoints in developing 21st century skills among the students.

8. Mathematics should be made inclusive as every other child to learn mathematics, and their needs (in terms of pedagogy, learning material etc) have to be addressed seriously. The conceptual world of mathematics can bring great joy to these children, and it is our responsibility not to deprive them of such education.

Recommendations and Suggestions for 21st century mathematics education

The learner needs to demonstrates understanding and appreciation of key concepts and principles of mathematics as applied - using appropriate technology - in problem solving, critical thinking, communicating, reasoning, making connections, representations, and decisions.

1.Perspectives about mathematical knowledge

Apart from developing mathematical ideas, emphasis should be laid on structure of mathematics, conceptual and procedural aspects of mathematical knowledge, content categories, process of generalization, pattern recognition, problem solving in mathematics- as a process & skill, mathematical investigations, verification or validation process of mathematical statements and role of critical thinking, creative thinking and historical perspective of mathematics.

2.Transactional Process: The transactional process has become very rigid and inflexible. Teachers need to be trained to use constructivist approaches and constructivist pedagogy to teach learning aspects related to mathematics according to Piaget, Bruner, Vygotsky, Dienes and Van Hiele.

3.Psychology of learning of mathematics

Designing learning environments for mathematics learning should be based on the learning theories and processes. Special reference should be made regarding mathematics learning at “Concrete operation stage” according to Piaget. Focus is

to be laid on the concrete operation stage of intellectual development of children at elementary level and “Formal operation stage” at Secondary level.

4. Appropriate use of Methodology/pedagogy

NEP -2020 emphasises that there should be more scope for experiential learning, collaborative and co-operative learning, blended learning and the classrooms have to adapt constructivist learning environments.

5. Development of 21st century Skills

21st century skills such as creative thinking, critical thinking, decision making, problem solving, reasoning, analysis, interpretation, synthesizing information, research skills and practices, interrogative questioning, creativity, artistry, curiosity, imagination, innovation, personal expression have to be developed among students through teaching of Mathematics.

6.Development of Integrated /holistic Curriculum

NEP 2020 recommends that the Mathematics curricula should aim for holistic development of learners, equipping them with the above mentioned 21st century skills, reduction in curricular content to enhance essential learning and critical thinking and greater focus on experiential learning. Students will have increased flexibility and choice of subjects so that they choose their own paths according to their talents and interests.

7.Use of mathematical Manipulatives: Teachers should use crucial aids such as black boards, charts, graph boards/charts, manipulatives, geo boards, geometry instrument box, calculators, computers etc. They should be well trained to make PowerPoint presentations, use internet sources, and use the computer operating systems and tools such as Ubuntu, Geogebra, Kturtle, kahoot, etc.

8.Assessment of mathematics learning

NEP 2020 recommends that there should be a shift from summative assessment to regular and formative assessment, which is more competency-based, promotes learning and development, and tests higher-order skills, such as analysis, critical thinking, and conceptual clarity by reforming the assessment and evaluation procedures by using a variety of assessment tools such as achievement tests, diagnostic tests, observation records, project work, seminar, discussions, checklist, student portfolios, self -assessment, and peer group assessment. Performance-based

assessment should be emphasized and a continuous and comprehensive evaluation should be focused.

9. Mathematical misunderstandings and Misconceptions

Teachers and students have many misunderstandings and misconceptions about mathematics. These have to be identified through assessments on conceptual understandings and clarified for effective new learning of mathematical ideas.

10. Special emphasis is laid on mathematical and computational thinking in NEP 2020.

“From age 6 onwards, computational thinking (the thought processes involved in formulating problems and solutions in ways that computers can effectively execute) will be integrated into the school curriculum. This is a fundamental skill in the digital age, and it can be effectively taught with well-designed paper worksheets... The school curriculum will offer optional subjects focused on programming and other advanced computer-based activities at the late upper primary and secondary stages.” [NEP 2019, P23.4.1]

“Integration of digital literacy: The new curriculum will also integrate digital literacy for all learners at the basic level, with hands-on assessments and worksheets keeping in mind the available digital infrastructure on the ground. At a more advanced level, curricula will be developed for: (i) Computational thinking; (ii) programming and other computer-based activities. Appropriate learning outcomes will be formulated as part of the National Curricular Framework in these subjects, and they will be extensively offered as courses in

middle and secondary schools with adequate computing and teacher resources.” [NEP 2019, P4.6.7.1]

4.Way forward:

4.1 Curriculum and Pedagogy

The Foundational stage

Early literacy and numeracy skills are not only life skills but also survival skills in this fast-changing world. Numeracy is the ability to recognise and apply mathematic concepts in all areas of life. It involves understanding of numbers, counting, solving number problems, measuring, estimating, sorting, noticing patterns, adding, and subtracting numbers and so on.

The need for strengthening early numeracy

In India most of the school age children are first generation learners and who may not have environment of literacy and numeracy at home.

Research has shown that once students fall behind on foundational literacy and numeracy, they tend to maintain flat learning curves for years, perpetually unable to catch up. The learning gap continues to widen up from that point as the text in the language textbook and mathematical concepts become more complex and abstract in later grades.

The National Achievement Survey (NAS) 2017 conducted for class 3,5,8 highlighted the problem of low learning levels which decline as the child progresses. Only 53% of students had acquired proficiency maths in class 3, which declined to 40% by the time they reach class 8. So, strengthening early numeracy skills in ECCE should be the top priority to support the students to achieve desired learning competencies at later stages.

In this context Ministry of Education has set up National Initiative for Proficiency in reading with Understanding and Numeracy (NIPUN) to ensure that every child in the country necessarily attains foundational literacy and numeracy in grade 3 by 2026-27 on priority.

Major focus of NEP 2020 is holistic development of the child. There are many interrelated and interdependent developmental domains and these domains have been subsumed into 3 major goals

- Developmental Goal 1 – Children maintain good health and wellbeing (HW)
- Developmental Goal 2 – Children become effective communicators (EC)
- Developmental Goal 3 – Children become involved learners and connect with their immediate environment. (IL)

Among these goals, Goal 3 (IL) is majorly focusing on cognitive and numerical skill development of children.

The major content areas of early mathematics are:

Pre-number concepts - These concepts are preliminary to the understanding of numbers and involves classification, order and set up one to one correspondence of objects and numbers to some extent. (Counts objects up to 100 in group of tens)

1. Numbers and operation on numbers –Numbers are the mathematical tools to count and measure. It Involves number sense (reading & writing numbers up to 999), reading of symbols, writing words and symbols, comparison of number like bigger than/smaller than etc... fundamental operations –addition (sum not exceeding 99), subtraction (upto99), multiplication (constructing and applying multiplication facts, tables of2,3&4), division (developing the idea of division) and their application in daily life.
2. Measurement – measurement is inherent part of human life. Thus, familiarity with different context of measurement is essential in our daily life. It involves length/distance, weight, capacity, time
3. Shapes and spatial understanding – It give children and Awareness of themselves in relation to people and objects. This content area includes shape, size, space, position, direction, movements,3 D shapes and solids, flat and curved surfaces of solids,2D shapes as seen on surfaces of a solid shape.eg – straight lines, curved lines, shapes made up of straight lines and curved lines etc.
4. Patterns –patterns are all around us. identification of patterns helps children to develop observation and analytical skills. It involves observing similarities, dissimilarities, repetition, non-repetition, growth/decay etc
5. Data handling—involves collecting, representing, and interpreting simple data, recording data using tally marks, collecting data, and representing in terms of pictograph choosing appropriate scale and unit for display through pictograph, drawing conclusion from the data
6. Mathematical communications – Language is essential to construct knowledge. Every discipline has special language. While sharing mathematical ideas and thoughts children should use mathematical language. It involves exchange of information between individuals through mathematical symbols, signs, diagrams, graphs.

NIPUN Bharat document shares detailed information about the above-mentioned content areas, competencies and learning outcomes, which can guide the construction of curriculum framework for Foundational stage of school education.

Pedagogical process to early numeracy:

NIPUN Bharat document recommends that "Quality foundational education should work on developing the important mathematical skills in children through the early mathematics concepts such as observation, reasoning, visualisation, generalisation, communication, critical thinking, problem solving, creativity and collaboration".

Suggestions for pedagogical approaches are:

- Learner centric pedagogy- Pedagogy should address the need of children. Equal focus should be given on child's conceptual development as well as problem solving skill development.
- Scope for exploration and mathematical thinking – encourage children to participate in activities and discussion, raising questions, making interpretations
- Use of manipulatives/toy pedagogy- devising as many different activities as possible with variety of materials is necessary. Concrete materials and toys should be used such as boxes, blocks, shape sorters, puzzles, dolls, toys etc.
- Mathematics with daily life – Children start learning numeracy skill from the time they are born. This learning happens by observing and experiencing numeracy in action. Especially in everyday play and activities. Daily activities like counting, talking about time and money, observing shapes and patterns can help children develop early numeracy and mathematic skills. Classroom process should give scope for this.
- Integrating mathematics with other subjects – It makes mathematics learning easy and develops application skill of maths in day today life.
- Joy in mathematics (recreation with mathematics) – Learning maths should be joyful experience for young learners. Mathematical concepts to be learnt through experiential method like hands on activities, games, riddles etc.
- Opportunities for peer learning, collaborative, and cooperative learning – It helps children to develop conceptual understanding and mathematical communication without any fear or hesitation.

The Preparatory Stage

NEP 2020 document recommends that, "It will build upon the play, discovery, and activity-based pedagogical and curricular style of the Foundational Stage, and will also begin to incorporate more formal but interactive classroom learning, in order to lay a solid groundwork across the subject mathematics".

The content and objectives of mathematics teaching at this stage are:

- develop skills in counting and understanding 3, 4 and 5-digit numbers.
- perform operations on 3, 4 and 5-digit numbers
- understand the standard algorithms of addition, subtraction, multiplication

and division of whole numbers.

- learn fractions and decimals and use them in real life situations.
- perform operations on fractions and decimals.
- understand 2-D and 3-D Shapes and their properties.
- understand the standard units of length, weight and volume.
- perform simple mathematical operations involving length, weight, volume and money.
- collect data and represent it in tabular form; draw pictograph, bar graph for given data.
- identify symmetrical shapes and patterns and make simple designs using shapes and patterns.
- identify number patterns such as patterns in odd and even numbers, triangular and square numbers, patterns in multiplication and division of numbers.
- develop Problem Solving skills to aptly use in real life situations
- develop pride about Indian Mathematicians contributions.
- develop interest in learning mathematics with ease and fun.

The pedagogical approaches should be based on experiential learning involving activities, games, puzzles, stories, dramatisation, Puppet show, activities through paper cutting (origami), drawing, Tangrams etc.

The Middle Stage

The middle stage mathematics teaching should attempt at the development of mathematical understanding and thinking in the child. It should emphasise the need to look at this middle stage as the stage of transition towards greater abstraction, where the child will move from using concrete materials and experiences to deal with abstract notions. It has been recognized as the stage wherein the child will learn to use and understand mathematical language including symbols. There is an

emphasis in taking the child through the process of learning to generalize, and also checking the generalization.

The content and objectives of mathematics teaching at this stage are:

- Understand the concept of whole number, integer, and rational numbers.
- Apply different methods in order to find the squares, cubes, square roots and cube roots of a given number.
- understand the concepts of ratio and proportion and use them to solve real life problems.
- understand the concept of variables and solves puzzles and daily life problems using variables.
- perform basic operations on algebraic expressions.
- represent daily life situations in the form of a simple equations and solve them.
- Identify number pattern and Observe patterns using algebraic operations in order to derive the divisibility rules of 2,3,4,5,6,9 & 11.
- Collect and arrange data and interpret pictograph, bar graph and pie chart.
- Calculate mean, median and mode.
- Use exponential form of numbers and apply rule of exponents to solve problems in real life situations.
- understand geometrical ideas like line, line segment, open and closed figures, angle, triangle, quadrilateral, circle, and their properties.
- find and estimate the area of different geometrical shapes.
- Identify and represent 3D objects, identify vertices, faces, edges.
- Finds surface area and volume of cube, cuboid and cylinder.
- Construct geometrical figures using the instruments.

The Secondary Stage

NEP 2020 has stated, The Secondary Stage will comprise of four years of multidisciplinary study, building on the subject-oriented pedagogical and curricular style of the Middle Stage, but with greater depth, greater critical thinking, greater attention to life aspirations, and greater flexibility and student choice of subjects.

At this stage, mathematics should be presented as an academic discipline to students. The students should perceive the structure of mathematics and hence the notions of abstraction, generalization, argumentation and proof should become central to mathematics curriculum.

NEP 2020 has recommended that "- - - students would continue to have the option of exiting after Grade 10 and re-entering in the next phase to pursue vocational or any other courses available in Grades 11- 12".

Hence, mathematics curriculum for Secondary school education should be framed for general course from 9th to 12th and specific mathematics curriculum required for particular vocational course.

The overall purpose of Mathematics Teaching and Learning at Secondary stage can be visualized as follows:

- know and appreciate the usefulness, power and beauty of mathematics through its patterns and structure.
- recognize that mathematics is all pervasive in the world around us.
- develop patience and persistence when solving mathematical problems.
- understand and be able to use the language, symbols and notation of mathematics.
- develop mathematical computation skills and thinking in mathematics.
- develop problem solving ability among students.
- develop mathematical curiosity and use reasoning and thinking when solving problems.
- become confident in using mathematics to analyse and solve problems both in school and in real-life situations.
- develop ability to integrate mathematics with other subjects.
- develop logical proofs for propositions in algebra, geometry and trigonometry.
- develop mathematical models for real life situations and problems.
- develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics.
- develop abstract, logical and critical thinking and the ability to reflect critically upon their mathematical work and the work of others.
- develop axiomatic approach to geometry.
- develop scientific thinking/temper and scientific attitude.
- appreciate the use of information and communication technology in mathematics.
- know and appreciate the contributions of Indian mathematicians to this filed since ages.

Content

NEP 2020 has recommended - - “Students will be given increased flexibility and choice of subjects to study, particularly in secondary school---“.

In this context, mathematics curriculum for secondary school education has to be framed at two levels - general and optional level for IX and X Standard.

Optional level mathematics curriculum should be characterised as follows:

- Topics in the curriculum for optional level should be in depth with rigour of mathematics.
- Practical and logical proofs are to be given for Theorems in Geometry with equal weightages to Converse Theorems and Corollaries.
- Good number of varieties of graded applications, integrating concepts/problems with other subjects like Physics, Geography etc.

For general level only practical proof, for a very few Theorems, without discussing Converse, Corollaries with a few simple, direct/numerical applications are to be given.

It is worth to discuss each ‘Constructions ’at the end of respective theorems and concepts with justifications for optional level & a few simple constructions without justification for general level.

Higher Secondary: (Class 11 and 12)

It is believed that students who did optional Mathematics at IX and X standard have already developed the skill of analytical thinking and logical reasoning, they also have developed the problem-solving skills. These skills are very essential for them to take up Mathematics at higher Secondary level (class 11 and 12).

At the higher secondary level, learners are expected to understand abstractness, develop the concepts using different approaches and try to find interrelation between the concepts. For example, solution of simultaneous linear equations can be interpreted geometrically to understand existence / nonexistence of solution, understand the interrelationship between the concepts and thereby find alternate solution to a mathematics problem, develop the processes of dealing with greater abstractions, moving from particular to general and vice versa.

There are several constructive approaches which are also recommended in NEP 2020 like - Experiential learning, inquiry-based, discovery-based, discussion-

based, and analysis-based learning. The mandated content will focus on key concepts, ideas, applications, and problem solving. Teaching and learning should be conducted in a more interactive manner; questions should be encouraged, and classroom sessions should regularly contain more fun, creative, collaborative, and exploratory activities for students for deeper and more experiential learning.

In conclusion, we wish to highlight NEP 2020 recommendation, “All curriculum and pedagogy, from the foundational stage onwards, will be redesigned to be strongly rooted in the Indian and local context and ethos in terms of culture, traditions, heritage, customs, language, philosophy, geography, ancient and contemporary knowledge, societal and scientific needs, indigenous and traditional ways of learning etc.– in order to ensure that education is maximally relatable, relevant, interesting, and effective for our students. Stories, arts, games, sports, examples, problems, etc. will be chosen as much as possible to be rooted in the Indian and local geographic context. Ideas, abstractions, and creativity will indeed best flourish when learning is thus rooted.”

4.2. Integration of Indian knowledge Systems in Mathematics Curriculum

The National Education Policy 2020 has recommended that Indian Knowledge System should be integrated with the curricular content.

“Knowledge of India” will include knowledge from ancient India and its contributions to modern India and its successes and challenges, and a clear sense of India’s future aspirations with regard to education, health, environment, etc. These elements will be incorporated in an accurate and scientific manner throughout the school curriculum wherever relevant; in particular, Indian Knowledge Systems, including tribal knowledge and indigenous and traditional ways of learning, will be covered and included in mathematics, - - -, agriculture, engineering,- - -.

The NCERT Curriculum of 2001 had mentioned that one of the objectives of teaching mathematics at the secondary stage was to "develop reverence and respect towards great mathematicians, particularly towards the Indian mathematicians for their contributions to the field of mathematics."

The Curriculum Framework for School Mathematics Education at all stages of curriculum should integrate Indian Knowledge System, that is the contribution of Indian mathematicians, life histories of mathematicians and stories of mathematical insights at appropriate curricular points.

Folklore mathematics

In Karnataka good work already exists in Folklore mathematics. More than 17 surveys have been done by universities and other adhyana kendras. We have rich corpus of Folklore mathematics collection which may be chosen with due modifications to suit the present social conditions. Folklore mathematics in the form of puzzles, riddles and problems can be suitably included at each stage of school mathematics curriculum.

Mathematics through origami

Mathematics learning can be made easy using Origami Skills. Origami (Paper folding) is usually employed to give a practical experience of mathematical concepts. Origami can be further expanded to teach theorems, identities, solid Geometry also. Paper folding activities can be suggested at all stages to enhance concept understanding of mathematical concepts and processes, create interest and also provide recreational activities. We can instil pride of India among children by highlighting the historical background of origami in mathematics. The first contribution on use of origami in representing mathematical ideas was from Tandanam Sundar Row, who published a book "Geometrical constructions in paper folding" in 1895.

We have wonderful achievements of Indian mathematicians. 90% of them are in Siddhanta Texts-Astronomical Texts. Astronomy content has to be increased in science curriculum, then it will be possible to link them to mathematics content.

4.3. Integrating mathematics with other subjects, arts and values

One goal of Mathematics teacher is to produce a mathematically literate nation where people can use the concepts from the subject to solve real life problems. So, Integrating Mathematics with other subjects, Arts and Values is most important.

- The Integration of mathematics with other subjects and real-life situations makes students think about the 'real world'.
- It also makes students start to think about why things are and happen in a certain way, giving them a practical approach to learning and using mathematics.
- Integration also helps to answer the common question posed by many students 'Where and when are we ever going to use this?'
- Integration allows students to see the usefulness and importance of mathematics which there after enables them to develop new understandings and skills.

- It helps the students to apply their mathematical knowledge in other subjects.
- Current brain research points out that the human brain looks for patterns and interconnections as it's the way of making sense of things.

4.3.1 Here are some ideas to help integrate mathematics with other subjects.

History

- Write a report on Indian mathematics.
- Learn about the history of clocks.
- Learn how various cultures told time throughout history and write a report.
- Calculate the number of years between various events.
- Learn about the history of scales and experiment with different types of scales.
- Learn about the history of currencies.

Science

- Write the distance between the sun and each planet using exponential form.
- Explore the half-life of certain radioactive elements or the size of the bacteria
- and viruses using negative exponents.
- Analyze rainfall over a time period for a specific area and create chart or graphs.
- Use Mathematics to prove various laws of physics.
- Measure and collect scientific data and use graphs, charts, lists, tables etc to organize the data.
- Compare the speed of several animals on a bar graph.
- Track the Weather or temperature and create a bar graph.

Geography

- Plan a trip by land, sea or air - Map a starting point and destination - Decide on appropriate Transportation and determine a reasonable speed - Then calculate the distance that will be travelled and the time the trip will take. Prepare a post card of it.
- Calculate the distance between various cities, states and countries.
- Learn to read grid coordinates and find places on a map through the use of latitude and longitude.

- Use a map scale to determine the distance between two points on a map to learn about the connection between scale and actual distance.
    - Use blocks to create replicas of famous buildings monuments or structures. While building discuss mathematical concepts such as perimeter area and volume.

Social Science

- Draw Bar graphs comparing populations per capita Income, population density of various country.
- Figure out the percentage of culture that speaks Kannada.
- Learn about the different types of currency in other countries.
- Go through a newspaper and block out all of the Numerals or Number words on a page, Read the article without the Numerals or Number words and discuss why they are so important.
- Learn about the role trade and value play in Economics.

Languages

- Write an essay on History of Mathematics.
- Provide written explanations for solutions to math Problems.
- Read stories that include counting, math facts etc.
- Create a mathematics Journal.
- Write an explanation of a recently learned mathematical concept, as if one is explaining it to another student.
- Write a paragraph explaining a selected Graph.
- Write a poem on mathematical concepts, Geometrical solids.

Physical Education

- Compare the percentage of wins and lose of a sports team.
- Draw a Graph to sort data for Olympic games.
- Discuss the food pyramid. Then compare foods on the pyramid to food eaten.
- Keep a fact counting dairy. Calculate the number of calories from fat eaten in a week. Find the daily average of fat and compare a fat intake with other family members and create a graph to compare.
- Incorporate and discuss mathematical ideas while playing games.

- Weigh and measure several family members/peers and create a bar graph showing the results.

Everyday life situation

- Learn Importance of financial literacy using mathematics.
- Learn how to calculate sales tax, discounts etc.
- Weigh fruits and vegetables and calculate what the total cost will be.
- Learn about home loans and learn how they are calculating Interests.
- Cook a meal and learn about fractions, doubling recipes, calculating weights ratio and proportion etc.

4.3.2 Integration of Arts in Teaching-learning process of Mathematics

Mathematics teaching and learning can be very creative through arts. Art integrated learning is a teaching-learning model which is based on 'learning through the arts' and 'with the art'. It is a process where art becomes the medium of teaching – learning process, a key to understanding concepts within Mathematics.

Learning through art makes abstract mathematical concepts concrete, understandable and approachable. Students will be able to visualize what they learn. Learners explore creatively while building connections between different concepts through various forms of art experiences. Both visual arts (Drawing, painting, clay modelling, pottery, papercrafts, masks and puppet making, heritage crafts etc.) and performing arts (music, dance, theatre etc.) lead to a better understanding and construction of knowledge about different concepts in Mathematics.

When students learn through arts, they go through different stages such as observing, thinking, imagining, exploring, experimenting, deducing, creation, recreating and expressing. These stages need actual involvement of all three domains namely cognitive, psycho-motor and affective. It is experiential in nature and leads to holistic development of every student.

For example, deriving a formula using an art activity provides creative space to every student to explore, experience and express without worrying about the judgement which helps them to overcome the subject fear and enhances their joy of doing and learning.

The important result of art integration into mathematics teaching is student's increased motivation and curiosity in learning. Art Integration in mathematics not only increases the curiosity, but aids in constructing a deeper understanding of Mathematical concepts. It helps children to develop creative problem- solving skills.

The other advantages of integrating Art in Mathematics are - Helps visualization, Connects with the real world, Inculcates respect for culture and heritage, Shifts from rote methods of learning, Develops art based enquiry, promotes Inter-disciplinary approach of learning and makes learning a joyful experience.

4.3.3. Integration of Values in Mathematics

There are number of questions like why should we teach mathematics? Why should everybody learn mathematics? A genuine teacher of mathematics will be interested in finding answer to these questions. One of the answers is that, it develops values among the children These values are essential for holistic development of children.

One of the principles of the National Education policy 2020 is, "The purpose of the education system is to develop good human beings capable of rational thought and action, possessing compassion and empathy, courage and resilience, scientific temper and creative imagination, with sound ethical moorings and values. It aims at producing engaged, productive, and contributing citizens for building an equitable, inclusive, and plural society as envisaged by our Constitution".

Some of the values that are developed through mathematics learning are:

Utilitarian values - Representing the surrounding mathematically, using special concepts (Spatial awareness) and number concepts in real life situations, applying mathematical ideas for modelling the real life problems and finding solutions, contribution to National Economy and Budgeting.

Disciplinary values - Simplicity, originality, Exactness, Independent, Linking to world, Verifying the results.

Cultural values, social values, moral values, aesthetic values, intellectual values and vocational values along with other values like art of Economic living (Time, speech, thought), power of Expression (Exact, precise, clarity, courage), self-reliance (Self- awareness), development of concentration, attitude of discovery, quality of hard work are also emphasised.

5. Assessment in mathematics

Assessment in the various school subjects provides the lenses which help to identify progress in attaining the aims of education which have defined the subject specific curricular expectations.

Assessment when used to support teaching and learning is a very effective means for communication between student, teacher, and parent. It serves as a navigation tool for all stakeholders to identify where they are and where they must reach. It flows naturally from the aims of education, the vision for teaching school mathematics and the mathematics curricular expectations. The purpose of this assessment is to systematically improve the quality of student learning through improved programs, curricula, and teaching (Gagne & Beard, 1978).

Competencies which stem from learning outcomes (which in turn may be traced back to curricular expectations and the aims of education) encompass both content and cognitive domains. When assessment is designed in alignment with the competencies, the performance of the students informs the teacher about the pedagogical practices and curricular materials needed to be set in place for the student to attain these competencies.

The hierarchical nature of mathematics requires frequent formative assessment since a learner who has difficulties in the fundamental concepts can often manifest inexplicable errors in later classes. Here, assessment plays a diagnostic role and feeds back into the learning process by identifying the strengths and weaknesses of both the learners as well as of the teaching. These can be addressed only by the rigorous tracking which formative assessment provides. Since students' performance in assessment for learning and assessment as learning is not graded, these assessments have very low stakes but high value for students.

Assessment must place priority on areas such as conceptual clarity and computational thinking. In the former, the learner should be able to recognise instances where the concept appears (and when it doesn't) and use the concept in a variety of contexts and situations. In the latter, mathematical tasks may be designed to identify whether a student can decompose a problem, recognise patterns, abstract important details or design, and use an algorithm.

While kindness dictates the use of familiar problems in examinations and tests, a learner may be truly enabled to solve problems if one learns the art of problem solving using such scaffolding.

The use of technology has been advocated in NEP 2020 but there is a risk of exclusion due to the digital divide. Rather than computer programming or coding, if students can be taught to compute, to find connections, to devise strategies and algorithms then they will be equipped to face the challenges of the 21st century. Assessment along these lines will ensure their preparedness for this.

The policy recommends that, "All subjects and corresponding assessments, beginning with mathematics, could be offered at two levels, with students doing some of their subjects at the standard level and some at a higher level". [NEP 2020, 4.38]

Assessment should be continuous and comprehensive and part of learning process. School based assessment using variety of tools and techniques like qualitative observation, anecdotal records, checklist, self-assessment, peer assessment, portfolio, interactions etc are preferable for all stages. Learning gaps should be identified and addressed immediately through various learning enhancement Programmes.

6. Textbooks and other Learning Resources

6.1 NEP 2020 recommends that, "All textbooks shall aim to contain the essential core material (together with discussion, analysis, examples, and applications) deemed important on a national level, but at the same time contain any desired nuances and supplementary material as per local contexts and needs". It further highlights that,

"All efforts will be made in preparing high-quality bilingual textbooks and teaching- learning materials for science and mathematics, so that students are enabled to think and speak about the two subjects both in their home language/mother tongue and in English."

- Mathematics textbooks should be presented in the most attractive manner for students.
- Content should be presented logically and in sequential form supported with examples, illustrations, figures, diagrams, anecdotes, biographies and stories.
- All mathematics textbooks should be made colourful to highlight the concepts.

- Thought provoking questions to be raised in the text for developing critical thinking and reasoning in students.
- Additional information and out of box ideas to be presented wherever possible.
- Variety of graded exercise to be provided in mathematics textbook.
- Questions should be related to real life context and also encourage higher order cognitive skill.
- The content must incorporate elements of values, conservation and sustainability, equity and inclusion while preserving the rigour of mathematics.
- Problems related to social issues, current real life situations and challenging situations to be presented.
- Workbooks, activity books and supplement textbooks for students; and teacher manuals and source books for teachers to be prepared in English and local languages.

6.2. ICT based learning is inevitable in education for 21st century. Hence, there is need to develop and encourage the use of digital content in teaching and learning of mathematics. Such digital content should be contextualised to the local situation and be made available in local dialects.

6.3. Every school should have a functional mathematics laboratory and mathematics club. District and State level mathematics fairs should be organised from time to time. Projects and activities done in schools can be shared in teacher forums and on digital platforms to enhance peer learning.

7. Teachers

Mathematics teachers play an important role in the whole learning process. Acquisition of quality pedagogical content knowledge is a must for every teacher. They must be flexible to meet the needs of the students in a creative and appropriate way.

Teachers should be offered regular and continuous professional development programs in mathematics content, pedagogy, teaching-learning materials and assessment procedures for mathematics.

Teachers should also be encouraged to take up action research in mathematics learning at the school level.

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