REVISION OF KARNATAKA STATE D.Ed. CURRICULUM

POSITION PAPER

KARNATAKA STATE SUB-GROUP

ON

ELEMENTARY MATHEMATICS TEACHER EDUCATION(*)

This is a draft position paper on "Elementary Level Mathematics Teacher Education" which is apart of pedagogic studies in the D. Ed. course. It is prepared by one of the sub-groups under the D.Ed. Curriculum Review Committee. This has to be discussed and further restructured after taking the views of all concerned stakeholders. **Reviews, Opinions and suggestions are welcome from all sections of stakeholders.**

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3. Executive summary

School education and teacher education are highly interrelated and interdependent. The goals, curricular areas, pedagogical approaches and evaluation procedures of teacher education are to be designed based on the conceptual and curricular framework of school education. This position paper attempts to review the various aspects of mathematics education in the light of NCF for School Education 2005 and RTE act 2009. Further, the paper analyses their implications on Elementary Level Mathematics Teacher Education. This paper also looks at the present scenario of Elementary Mathematics Teacher Education, presents a critique on the issues and makes recommendations for desirable changes.

Mathematics school education

Mathematics is a compulsory core subject of study at school level. It's main goal is **"mathematisation of child's thought processes".** In order to realise this goal, NCF (2005) document has prescribed the curricular areas for elementary level mathematics. In Karnataka State context, already the syllabus is revised on par with NCF 2005, and is being implemented from June 2012. Apart from NCF (2005), two other factors namely, Universalisation of Elementary Education and RTE Act (2009) have greatly influenced elementary school mathematics education. Despite the changes taking place in the mathematics education, it is identified with several problematic issues. One of the major issues is the "lack of teacher preparation and support in the teaching of mathematics".

In this context, NCF for Teacher Education (2009) has called for an urgent and comprehensive reform in teacher education programmes. It has become imminent to review, revitalise and restructure mathematics teacher education at elementary level.

Elementary mathematics teacher education

Mathematics being a compulsory subject in school education, elementary level mathematics education and pedagogic studies on mathematics education has to become a compulsory paper of study in D. Ed curriculum. In setting up a new approach for mathematics teacher education programmes or revising the existing programmes, it is important to consider what that approach might involve. NCF

(2005) and NCFTE (2009) have recommended constructivist pedagogy and a shift from teaching to make students learn.

Specific concerns related to D. Ed. course

1) Student-teachers lack conceptual understanding of mathematics. Majority of humanities background student-teachers are opting for mathematics methods paper.
2) Content enrichment programme prescribed for 2 months in the first year of D.Ed. course is inadequate and cannot enable trainees to acquire mastery over the subject matter.
3) Mathematical misunderstandings among student-teachers and school children are another concern which is neglected and not taken care in the D.Ed. course.
4) Student-teachers might not have covered all the aspects of elementary mathematics curriculum and their conceptual understanding of covered areas may need to be questioned.
5) Mathematical knowledge of teacher educators is also found to be inadequate.
6) The problem of recycling of mathematical beliefs, attitudes and thoughts are a major concern. Not even minimum opportunities are provided for student-teachers to reflect on their own beliefs and modify them.
7) In the present D.Ed. course, transactional process has become very rigid and inflexible. Constructivist approaches to teach mathematics and constructivist pedagogy to train student-teachers are not focussed.
8) Student-teachers are adult learners and they need opportunities for reflections on their own experiences and assumptions. But the present D.Ed. classrooms are structured didactically and lecturing is the only mode of delivery.
9) Focus is not laid on the concrete operation stage of intellectual development of children at elementary level.
10) Constructivist approaches and learning aspects related to mathematics according to Piaget, Bruner and Vygotsky are not given due importance.
11) Student-teachers are not trained to use ICT tools in the teaching-learning process.

12) Assessment procedures followed in the D.Ed. course are not satisfactory and they do not assess the student-teachers' conceptual understanding of mathematics and performance levels of their teaching competencies.

Vision statement

Sensitisation of prospective teachers to,

- reflect on their own mathematical content taught at elementary level.
- construct their own pedagogical knowledge regarding mathematics.
- become a humane and professional mathematics teacher

Recommendations for realising the vision and the prescribed objectives are:

1) Knowledge base of mathematics teachers should be strengthened in the D.Ed. course. Major components of the knowledge base are,
   a) Content knowledge of mathematics which includes conceptual understandings and mathematical representations.
   b) Pedagogical content knowledge of mathematics which includes knowledge of learning and knowledge of teaching.

      Knowledge of learners should include, Knowledge of characteristics of learners with reference to their level of intellectual development – concrete operation stage, learning process and learning styles.

      Knowledge of teaching should include, Knowledge of goals of elementary mathematics education, approaches to content of mathematics, ways of representing mathematics to elementary level children and use of various resources and materials.

   c) Knowledge of organisation and management for teaching mathematics: This refers to the knowledge a teacher needs to organise Individual/group or whole class teaching, various activities, different styles of learning and teaching, and manage various resources.
   d) Knowledge of context of teaching
   e) Knowledge of education

This position paper suggests a model which is **"knowledge based"** elementary mathematics teacher education. This model can further include in its framework **"competency based and commitment oriented"** teacher education programme as recommended by NCTE and **"Process – based"** teacher education as proposed by NCFTE (2009).

2) Structure of D.Ed. course related to pedagogic studies

Learning of mathematics and mathematics pedagogy should be made an on-going activity during the D.Ed. course. Two possible models are discussed in this position paper.

A) If the D.Ed. course continues as a non-degree course (as in practice at present) study of all the three core subjects, their content knowledge as well as pedagogic knowledge can be designed for the 2 year D. Ed. Course as follows.

I Year - I language + EVS + Mathematics

II Year - English + opting any two subjects from Science/Mathematics/Social Science

During the I year content knowledge can be prescribed from I Std to IV Std mathematics school curriculum. Pedagogy of teaching this content should be included in paper. The paper can be titled as "Mathematics Education for Lower Elementary Level" (LEL). During the II year, content knowledge can be prescribed from V Std to VII Std mathematics school curriculum. Also pedagogy of teaching this content should be included in the paper titled "Mathematics education of upper Elementary Level (UEL)".

B) If the D.Ed. course is transformed into a "Degree Course" as suggested by NCFTE (2009), then provision should be made for student-teachers to opt for any two special subjects in accordance with the subjects they had studied at under-graduation level.

3) **Curricular areas related to content knowledge**

In addition to the already prescribed mathematics content in the current D.Ed. curriculum, topics such as data handling, patterns, 2D & 3D figures and symmetry should be included. Units on these content areas should be dealt from the point of view of content analysis and pedagogical analysis.

4) **Perspectives about mathematical knowledge**

Apart from meaning and nature of mathematics, emphasis should be on structure of mathematics, content categories, conceptual and procedural aspects of mathematical knowledge, historical perspective of mathematics knowledge, process of generalisation, pattern recognition, problem solving in mathematics- as a process & skill, mathematical investigations, verification or validation process of mathematical statements and role of critical thinking and creative thinking in mathematics.

5) **Mathematical misunderstandings**

There are many misunderstandings about elementary mathematics both in children as well as student-teachers. These have to be identified through assessments on conceptual understandings and clarified for effective new learning of mathematical ideas. Constructivist approaches should be followed to provide remedial treatment.

6) **Psychology of learning of mathematics**

Constructivist learning theory should be the central focal point in the D.Ed. curriculum. **Theories and processes of mathematics learning according to Piaget, Bruner, Asubal, Vygotsky, Dienes and Van Hieles** should be included. Designing learning environments for mathematics to be based on these theories and processes should be highlighted. **Special reference should be made regarding mathematics learning at "Concrete operation stage" according to Piaget.**

7) **Transactional Mode**

The catch word in Educational scenario at present is "**Constructivism**". NCF for school Education, 2005 and NCF for teacher education, 2009 have recommended **constructivist** approaches for mathematics learning as well as mathematics teacher education.

This has to be predominantly highlighted for transacting D.Ed. curriculum. D.Ed. classrooms have to inculcate constructivist learning environments for student-teachers to construct mathematics knowledge and pedagogical knowledge of mathematics. This will further enable them to design constructivist learning environments for mathematics in school level classrooms. The implications of constructivist approaches are that co-operative learning strategies and

collaborative learning should become the mode of transactional processes at both school level and D.Ed. level.

8) **Current practices in schools**

During the 1st year, focus should be on the on-going practices of teaching in lower elementary level. 'Nali Kali' practice based on constructive approaches should be mainly discussed. Student-teachers should observe nail-kali lessons, interact with mentors in schools, plan to handle nail kali lessons under the guidance of mentors as well as teacher educators and execute them during block practice.

9) **ICT in mathematics education**

In the D.Ed. course Student-teachers should be exposed to both low technology and high technology aids that can be used to teach mathematics. Some of the crucial aids that student-teachers should be able to use are black boards, charts, graph boards/charts, geo boards, geometry instrument box, calculators, OHP and transparencies, computers etc. They should be well trained to make PowerPoint presentations, use internet sources, and use the computer operating systems and tools such as Ubuntu, Geogebra, Kturtle, etc.

10) **Assessment of mathematics learning**

It is a well-known fact that undue importance is given to examination and results in the D. Ed course. This practice has to be discouraged by completely reforming the assessment and evaluation procedures. As constructivist approaches have to be the transactional mode in D.Ed. course as well as school education, assessment procedures should be accordingly implemented by using a variety of assessment tools such as achievement tests, diagnostic tests, observation records, project work, seminar, discussions, checklist, student portfolios,selfassessment,andpeergroupassessment.Performance-based assessment should be emphasised and a continuous and comprehensive evaluation should be focussed.

4. Introduction

School education and teacher education are like two faces of a coin and they are highly interdependent and interrelated. Any teacher- education programme is to be designed based on the conceptual and curricular framework of school education. The goals, curricular areas, pedagogical approaches and evaluation procedures of teacher-education are off shooted from school curriculum. This clearly indicates that **POSITION PAPER on Elementary Mathematics Teacher Education** should be discussed and framed in the light of curriculum of mathematics school education.

The curriculum of mathematics at elementary school level in Karnataka is revised and modified on par with the national curriculum. Two significant developments particularly, the NCF 2005, and the Right of children to free and compulsory education Act 2009 have guided the development of this curriculum on mathematics for Elementary School Education.

This position paper attempts to review the various aspects of mathematics education in the light of NCF for school education 2005 and RTE act 2009. Further, the paper analyses their implications on Elementary level Mathematics Teacher Education. This paper also looks at the present scenario of Elementary Mathematics Teacher Education, presents a critique on the issues and makes recommendations for desirable changes.

4.1 Goals of mathematics education

Mathematics is a compulsory core subject of study at school level. The position paper on NCF, 2005 has emphasised very well the main goal of mathematics education at school level i.e**. mathematisation of child's thought processes**. With regard to elementary level school mathematics the primary aim of mathematics education specifically relates to **numeracy**. The higher aim relates to developing the **child's inner resources such as thinking capabilities**. The kind of thinking the child learns in mathematics is an ability to handle abstractions and to solve problems. The NCF, 2005 document has prescribed the curricular areas for elementary level mathematics in order to realise the above mentioned goals of mathematics education at elementary level. In Karnataka State

context, already the syllabus is revised on par with NCF 2005, and is being implemented from June 2012.

Apart from NCF 2005, two other concerns which have great impact on mathematics education are, Universalization of Elementary Education (UEE) and Right to Education Act (RTE).

Two important implications of these on mathematics education are,

- Schooling is a legal right and mathematics being a compulsory subject, access to quality mathematics education is every child's right. That is, mathematics education should be affordable to every child and at the same time enjoyable.
- Eight years of elementary school mathematics should engage the mind of every student and strengthen the child's inner resources.

4.2 Problems in teaching and learning of mathematics

Despite the changes that are taking place in mathematics school education system and the strong recommendations made by various education commission reports and policy documents, it is observed that mathematics education at elementary level is identified with several problematic issues. The position paper on teaching of mathematics has enumerated the following four problems to be the **core areas of concern.**

1. A sense of fear and failure regarding mathematics among the majority of children.
2. A curriculum that disappoints both talented minority as well as the non-participating majority at the same time.
3. Crude methods of assessment that encourage perception of mathematics as mechanical computation.
4. Lack of teacher preparation and support in the teaching of mathematics.

The fourth area of concern has been expanded further related to elementary level mathematics teacher education.

At the primary level most of the teachers assume that they know all the mathematics needed and in the absence of rigorous pedagogic training simply try and uncritically reproduce the techniques they experience in their school days. According to Wilson (1987)"**while a personal understanding of the subject matter**

may be necessary, it is not a sufficient condition for being able to teach". Text book centred pedagogy dulls the teacher's own mathematics activity.

The main consequence of this is "**mathematics pedagogy rarely resonates with the findings of child's psychology".** A critical analysis of mathematical education system in Karnataka reveals that, it is largely guided by the rigid mathematics syllabus, teacher centred pedagogy and examination system. A marked feature of mathematical educational practices in schools is a dull, routine, and rote system of learning.

4.3 RECOMMENDATIONS

In order to realise the goals of mathematics education at elementary level and overcome the problems the recommendations made are as follows:

- Shifting the focus of mathematics education from achieving primary goals to higher goals i.e., **a shift in focus from mathematical content to mathematical learning environments**, where mathematical processes are given importance. These mathematical processes are problem solving, use of heuristics, estimation and approximation, use of patterns, visualisation, and mathematical communication.
- Engaging every student with a sense of successes while at the same time offering conceptual challenges to the emerging mathematician.
- Changing modes of assessment to examine student's mathematisation abilities rather than procedural knowledge.
- Enriching teachers with a variety of mathematical resources.

As per the vision of position paper on teaching of mathematics excellent mathematics education is based on twin premises that **all students can learn mathematics and that all students need to learn mathematics.** This is the visualization of the place of mathematics teaching in the curricular frame work of elementary school mathematics education.

In this context of UEE, RTE 2009, and NCF-2005 for school education there is great demand on the role of teachers in enhancing the quality of mathematics

education. **A number of factors may influence the learning of mathematics but teachers play an important role in the learning process.** The common belief is that, if a teacher knows mathematics very well he/ she is the best person to make students learn mathematics. But, what about **'knowing to enable students to learn mathematics'?** There is a need for a fundamental change in the approach to the preparation of elementary school mathematics teachers. **The NCF on teacher education 2009 has called for an urgent and comprehensive reform in the teacher education programmes in general and also in elementary level teacher education programmes**. It has become imminent to review, revitalise, and restructure teacher education at elementary level.

5. **Elementary mathematics teacher education**

The professional preparation of teachers is crucial for the qualitative improvement of school education. According to Yashpal committee report (1993) on **Learning without burden** "Inadequate programmes of teacher preparation lead to unsatisfactory quality of learning in schools. The content of the programme should be restructured to ensure its relevance to the changing needs of school education. The emphasis, in these programmes should be on enabling the student-teacher to acquire the ability for self-learning and independent thinking." Therefore in pre-service elementary teacher education programmes stress to be laid on realising the vision of quality elementary school education. This specifically refers to elementary level mathematics education also. **The goals of mathematics education, its concerns, problems, and recommendations, as already discussed have an extended impact on pre-service elementary teacher education. Mathematics being a compulsory subject in school education, elementary level mathematics education and pedagogic studies on mathematics education has to become a compulsory paper of study in D. Ed curriculum.** Another reason for compulsory place of the paper "Elementary Level Mathematics Education" in D.Ed. curriculum is that every student-teacher is expected to teach mathematics after recruitment. The implication of this aspect is that the pre- service elementary teacher education programme should aim at training every student teacher to teach quality mathematics.

The questions to be answered at this junction are,

- What should be the quality of elementary level mathematics teacher?
- What mathematics content should they understand and construct?
- What pedagogic training should be given for them?
- What aspect of child psychology should they know to resonate mathematics pedagogy with it?

  This position paper makes an attempt to analyse the existing problems and concerns related to elementary mathematics teacher education, explore the possible answers for the above raised questions and recommend a curricular frame work for quality elementary mathematics teacher education programme, which enables the student-teacher to realise the vision of elementary mathematics education. NCF 2005 has recommended **constructivist pedagogy** and this has altered the traditional teacher centred instructional practices. The major shift is from ***'teaching'*** **to making *children learn*. Teachers are expected to facilitate constructivist learning environment for children to construct their own knowledge.**

  The sub group's reflection on the place of elementary mathematics teacher education in D. Ed curricular frame work is essentially positioned on the two aspects:

  - Provide help to the student-teacher in becoming reflective practitioner who learns from his/her own experiences.
  - Emphasise learning with understanding and learning to learn.

It is therefore, essential to answer the following two questions to justify the curriculum frame work of "Elementary Mathematics Teacher Education" paper in the D.Ed. curriculum.

1) What opportunities can be provided by the papers"Mathematics education for the elementary child" and "elementary mathematics education" for theStudent-teachers to
   - Observe and engage with children
   - Communicate mathematics with them.
   - Relate mathematics to children

2) How it can

- Strengthen their self-learning, reflection, assimilation and articulation of new mathematical ideas.
- Develop capacity to think and self-directed learning.
- Guide them to work in groups.

In setting up a new approach for mathematics teacher education programme or revising the existing programme, it is clearly important to consider what that approach might involve. In other words, what the programme might contain for mathematics and student-teachers embarking on a two year long D.Ed. course in the particular context of classroom related issues in mathematics education.

5.1 Specific concerns of elementary mathematics teacher education

NCF for teacher education 2009 has noted that "Initial training of elementary teachers continues to suffer from isolation, low profile, and poor visibility, in view of it being non-degree programme".

It's recommendations are,

- Up-grading initial teacher education by enhancing the entry qualification and duration of training making it equivalent to a degree programme and locating the management and control of elementary teacher education within universities.
- Up grading elementary teacher education calls for participatory curriculum planning involving all stake holders, modular organisation of curriculum in terms of critically engaging with theory and bringing practice with in its perspective and professional approach to teacher education processes.

In the light of this background, **specific concerns** related to training provided to student-teachers in the current D. Ed. course are discussed below.

1. It is assumed that student-teacher would have,
    - a secure knowledge of mathematics,
    - acquired a rich conceptual structure of mathematics,
    - been familiar with the range of mathematical topics in school curriculum.

But, several research studies have revealed striking results that, **student teachers lack conceptual understanding of mathematics and ability to express mathematical concepts and relationships.**

- In the existing D.Ed. Programme in Karnataka State majority of science students opt for English and Science methodology and majority of humanities students who are less interested in mathematics, are left with no other alternatives but to opt for Social Science and Mathematics education papers as their methodology. Hence **their mathematics knowledge which is limited to their learning of mathematics at school level up to 10th standard is highly inadequate.** Another added concern to this issue is **'mathematical knowledge' of teacher educators who train the student teachers in the D.Ed. course.**

  "Pre - Service Elementary Teacher Education in Karnataka:A Status Study" states that 75% of the teacher educators' content and methodological (pedagogic) knowledge is in between 25% to 50%.
- The third dimension of this problem is that **"content enrichment programme"** is prescribed for only two months in the 1st year D. Ed. Course. This is highly inadequate and does not justify the important expectation that student teacher should have **mastery over the content knowledge of elementary mathematics**. Moreover, this two months content enrichment programme has become **more of a mechanical process than laying emphasis on conceptual understanding of the subject matter**.

2. Another point of serious concern, which is almost neglected and not to be found in the current D. Ed. curriculum is related to **'mathematical misunderstanding'.** Research findings have reported that mathematical misunderstandings present among school students are also found to exist in substantially the same proportions among student-teachers.

3. One of the major concerns for pre-service mathematics teacher education is the **dispositions, beliefs and attitudes student-teachers have towards the subject mathematics and teaching of mathematics.** Teacher beliefs, knowledge, judgements and thoughts have an effect on the decisions they make which influence their plans and actions in the class room. It is evident that the quality of

mathematics learning of students is positively correlated with the attitudes of teachers towards mathematics and teaching of mathematics.

A review of the present D.Ed. curriculum shows that **not even minimum opportunities, experiences and interactions are created for the student-teachers to question and reflect on their own beliefs, modify them and develop required beliefs and thoughts.**

The new entrants' attitudes and aptitudes towards the subject mathematics and teaching of mathematics are rarely assessed and the attempts made to assess or enhance them during the D. Ed. course are inadequate.The problem of recycling of mathematical attitudes is a major concern and it is very resistant and resilient to change.

4. Apart from the student-teachers having misconceptions about mathematics they have studied, they may also have not covered (studied) all aspects of the elementary mathematics curriculum themselves. It cannot be assumed that student-teachers have 'covered' all aspects of the elementary mathematics school curriculum and, in addition, the **conceptual understanding of the covered areas may need to be questioned.**

5. In the present D. Ed. course, transactional process has become very rigid and inflexible. It tends to suppress the natural curiosity and creativity of student-teachers. One of the main reasons for this may be the dominating role of theory based examination system**. Primary importance is not given to construct theoretical ideas and develop pedagogical skills.** This is a serious concern and cannot be ignored. The issues concerning this are highlighted in many Education Commission Reports since 1964, NCF (2005), NCFTE (2009) and Dr.Venkateshmurthy's report. Some of the highlights are presented below.
   - It lacks peer group discussion, interaction and reflective practices.
   - The heterogeneous nature of student-teachers is not taken into consideration while transacting the curriculum.
   - Active participation in mathematics learning and pedagogic studies is not given primary importance.

- The constructive approaches to teach mathematics and the constructive pedagogy to train student-teachers as propagated by the NCF, 2005 and NCFTE, 2009 are not focussed in the present curriculum. And also there is a dearth of expertise in modelling constructive strategies as per their expectations.

These issues are further endorsed by the observations reported in the Pre-Service Elementary Teacher Education in Karnataka: A status study by Karnataka Jnana Aayoga.

- Planning for a constructivist classroom is non-existent. Classroom environment, without exception, is conducive for lecturing.
- Lessons are structured didactically and lectures are the only mode of delivery. All classrooms are devoid of resources.
- There is minimal and perfunctory interaction among teacher educators and student-teachers.
- As to assessment, none of the D. Ed classrooms model technique of formative assessment which elementary school teachers are expected to use.

The report concludes that, the D. Ed. classrooms are thus a far-cry from what is expected of teachers in elementary schools. While a classroom populated by adults is not expected to be completely hands-on, opportunities for participation and interaction are necessary, if future teachers are expected to emulate them. Without adequately engaging with the D. Ed. students' beliefs and attitudes the theory that is being drilled into them may be of little value.

6. Student-teachers in the D. Ed. course are "adult learners". As "adult learners" they need to be provided spaces and opportunities for reflections on their own experience and assumptions. But in reality, current D. Ed course classrooms are structured didactically and lectures are the only mode of delivery. NCF, 2005 and NCFTE, 2009 have recommended **'Constructivist approaches'** to make dialogues and reflective teaching to be the norm in D. Ed. curriculum. As long as this issue is not resolved and taken care of, **'mathematical learning' 'mathematical teaching'**

and 'mathematics teacher education' at elementary level will always take the back seat in the system.

7. Teachers are the key to provide effective elementary mathematics education. In order to do this, their knowledge base forms the sound foundation. It is a well-known fact that elementary level mathematics teachers' knowledge base is rather very bleak.

   This is a major concern and the questions to be answered to resolve this issue are,

   - What should be the knowledge base of elementary level mathematics teachers?
   - How should this knowledge be transacted in the D. Ed. course?

8. A cursory glance at the mathematics pedagogy paper of the current D.Ed. curriculum reveals the gaps required to be filled up, if we have **to sensitise student-teachers and future teachers to cope up with the curriculum changes in mathematics at school level.** Some of the major issue points are listed below.

   a) Topics listed in "content enrichment paper" are drawn from elementary mathematics curriculum. But it does not concur with the revised curriculum on par with NCF (2005). Topics such as Mental Arithmetic, Patterns, Date handling, 2D and 3D figures should be included in D. Ed. level mathematics curriculum.

   b) The paper "content-cum-methodology of teaching mathematics" is very exhaustive in nature including all the aspects of mathematics teaching. Teacher educators at D.Ed. level are of the opinion that it is too overburdened and cannot be completed in the prescribed duration. The reason is student-teachers go for Block practice teaching for 40 days and internship for three months. Another added problem is that annual exam for 2nd year D. Ed students is quite often pre-poned by either 1 or 2 months. Hence, few months are left to cover the entire theory portion of this paper. These reasons hinder the learning of methodology of teaching of mathematics by student-teachers. A possible alternative solution has to be worked out in the revised curriculum.

   c) Another glaring concern to be discussed is about the **psychology of learning of elementary level children.** Even though contribution of Piaget and Bruner is

included in unit 1 of the paper, in reality majority of the teacher educators do not discuss it, assuming that it is already covered in psychology paper. Those who teach it, simply discuss the four stages of cognitive development according to Piaget and three stages of learning process according to Bruner. Unfortunately, the focus is not laid on two important factors which are discussed below.

- Children at elementary level are at the **"concrete operation stage" of intellectual development.** Their characteristics and learning capabilities should be particularly highlighted and student-teachers should develop a clear understanding about these ideas. Only then can they plan the learning activities and environments to communicate mathematical ideas to children. This aspect is not highlighted in the current Content cum methodology of teaching mathematics Paper of D. Ed. curriculum.
- Piaget's and Bruner's theories of learning are not only general, but specific contributions are made related to learning of mathematics. First of all, mathematics curriculum for elementary level should be developed based on these notions and learning of mathematics should be structured in resonation with the capabilities of children to learn mathematics at concrete operation stage. This should form the basic principle of constructing curriculum for elementary level mathematics teacher education. A detailed review of the current D. Ed. curriculum and various reports on the status study of D. Ed. curriculum in Karnataka shows that mathematics education is not dealt form this dimension.
- Another missing element is **Vygotsky's theory of social learning.**

  These two aspects have to be prominently highlighted in the revised curriculum. **Only then we can justify that constructivist pedagogy is included in mathematics education.**

Against this background, the vision and objectives of "Elementary Mathematics Teacher Education" are stated in the section below:

6. Vision statement

Sensitisation of prospective teachers to,

- Reflect on their own mathematical content taught at elementary level.
- Construct their own pedagogical knowledge regarding mathematics.
- Become a humane and professional mathematics teacher

7. Objectives: To enable student-teachers to,

a) view learners as active participants in their own learning and not as mere recipients of knowledge
b) encourage the capacity of learners to construct the knowledge
c) Ensure that learning shifts away from rote methods.
d) View learning as a search for meaning out of personal experiences and
e) Knowledge generation as a continuously evolving process of reflective learning.
f) Organise learner-centred, activity- based, participatory learning experience – play, projects, discussion, dialogue, observation, visits, integrating academic learning with productive work.
g) Engage with the curriculum, syllabi and textbooks; to critically examine them rather than taking them as 'given' and accepted without question.
h) Create joyful learning environment to learn mathematics.
i) Develop understanding in the appropriate usage of terms, concepts, symbols, algorithms, mathematical techniques.
j) Communicate in mathematical language.
k) Provide an opportunity to make up interesting problems and create new dialogue in mathematics.
l) Develop logical thinking.
m) Appreciate the scope and power of mathematics
n) Adopt collaborative learning technique.
o) Use Information and Communication Technology tools in providing learning experience in mathematics teaching at elementary level.

What are the implications of the above discussed concerns and objectives for pre-service elementary mathematics teacher education?

What should be the knowledge base of student-teachers of D. Ed. course?

8) RECOMMENDATIONS FOR D.Ed. COURSE

1. Knowledge base of mathematics teachers

The recommended components of mathematics teacher's knowledge are:

- **Content knowledge (subject matter knowledge) of mathematics:**

This includes both the knowledge of mathematics and knowledge of mathematical representations. Knowledge of mathematics is about, conceptual understanding of topics of elementary level school mathematics.

Knowledge of mathematical representations is emphasised, because mathematics is seen as a composition of large set of highly related abstractions. Fennema and Frank (1992) state that "if teachers do not know how to translate those abstractions into a form that enables learners to relate mathematics to what they already know, they will not learn with understanding".

- **Pedagogical content knowledge (knowledge of teaching mathematics):**

Pedagogical content knowledge of mathematics is the knowledge which a teacher uses to transform and represent knowledge of mathematics for teaching (Wilson-1987). This refers to the ability of the teacher to transform content into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the learners. Based on this notion of pedagogical content knowledge, effective teachers can possess an in-depth knowledge of how to represent the subject matter to learners. This includes knowledge of learners and knowledge of teaching. **Knowledge of learners** should include,

- Characteristics of learners with reference to their level of intellectual development.
- Knowledge of learner's learning strategies and learning process.

Knowledge of teaching should also include,

- Knowledge of goals of elementary mathematics education.
- Knowledge of approaches to content of mathematics.
- Knowledge of ways of representing mathematics to elementary level children.
- Knowledge of use of various resources and materials.

In order to construct mathematical concepts in student's mind, pedagogical knowledge as well as mathematical content knowledge is needed. They are the integrated parts of effective mathematics instruction. The manner in which teachers relate their subject matter (what they know about what they teach) to their pedagogical knowledge (what they know about teaching) and how subject matter knowledge is a part of the process of pedagogical reasoning are seen as integral parts of pedagogical content knowledge.

There are some more supporting knowledge bases of teaching mathematics at elementary level.

- **Knowledge of organisation and management for teaching mathematics**. This refers to the knowledge a teacher needs to organise
  - Individual, group or whole class teaching;
  - Various activities,
  - Different styles of learning and teaching,
  - And manage various resources.
- **Knowledge of context of teaching**

  It is described as the knowledge base required by a teacher as one in which they can perform effectively in teaching mathematics to a class.
- **Knowledge of education**

  This is the knowledge related to educational issues which have immediate and obvious relevance for the beginning elementary mathematics teacher. There are a range of diverse issues which influence mathematics learning such as, special needs of children learning mathematics, ethnicity and gender, recently documented concerns relating to National curriculum Frame work etc.

This position paper suggests a model which is **"knowledge based"** elementary mathematics teacher education. This model can further include in its framework **"competency based and commitment oriented"** teacher education programme as recommended by NCTE and **"Process – based"** teacher education as proposed by NCFTE (2009).

2. Structure of D.Ed. course related to pedagogic studies

Content enrichment programme is for 2 months in the current D.Ed. course. It is very unrealistic to assume that the entire elementary level mathematics content knowledge can be revised during this period. Even though this paper is common to all student-teachers, only a part of them get training in teaching mathematics in the second year.

The revision of D.Ed. curriculum should take care of this deficiency. This position paper suggests that learning of mathematics and mathematics pedagogy should be made an on-going activity during the D.Ed. course.

Two possible models are discussed in the following section.

A) If the D.Ed. course continues as a non-degree course (as in practice at present) study of all the three core subjects, their content knowledge as well as pedagogic knowledge can be designed for the 2 year D. Ed. Course as follows.

I Year--- I language + EVS + Mathematics

II Year--- English + opting any two subjects from Science/Mathematics/Social Science

During the I year content knowledge can be prescribed from I Std to IV Std mathematics school curriculum. Pedagogy of teaching this content should be included in paper. The paper can be titled as "Mathematics Education for Lower Elementary Level" (LEL).

[Note: i) If 5th Standard is shifted to LEL, then the content prescribed for that level in school curriculum can be added to the paper for I year.

During the II year, content knowledge can be prescribed from V Std to VII Std mathematics school curriculum. Also pedagogy of teaching this content should

be included in the paper titled "Mathematics education of upper Elementary Level (UEL)".

ii) If VIII Standard is shifted to UEL, then the content of VIII Std school mathematics should be added to the paper for II year.

iii) The same pattern can be adopted for I language English, Science, EVS and Social Science to maintain uniformity.

iv) A paper on "English proficiency" should be made compulsory for all during the Ist year.

B) If the D.Ed. course is transformed into a "Degree Course" as suggested by NCFTE (2009), then provision should be made for student-teachers to opt for any two special subjects in accordance with the subjects they had studied at under-graduation level.

3) Curricular areas related to content knowledge

In order to realise both the primary aim and higher aim of teaching mathematics, student-teachers should be equipped with thorough content knowledge of school mathematics.

Content areas to be included are numbers, fractions, decimals, percentages, Ratio and proportion, concepts and procedures related to mathematics in daily life activities (Profit & loss, discount, Interest etc.), mental arithmetic framing and solving simple linear equations, space and shape, measurement, Data Handling, Patterns, 2D & 3D figures, symmetry and constructions in geometry. Units on these content areas should be dealt from the point of view of content analysis and pedagogical analysis.

4) Perspectives about mathematical knowledge

Apart from meaning and nature of mathematics, emphasis should be on structure of mathematics, content categories, conceptual and procedural aspects of mathematical knowledge, framework for acquisition of concepts with respect to mathematical knowledge, and effect of socio-cultural background of children on

mathematical knowledge. It should also include historical perspective of mathematics knowledge. In order to cater to the higher aim of teaching mathematics, focus should be on mathematical reasoning. Points to be included are inductive reasoning, process of generalisation, pattern recognition, problem solving in maths- as a process & skill, mathematical investigations, verification or validation process of mathematical statements and role of critical thinking and creative thinking in mathematics.

5) Mathematical misunderstandings

Students do not come to the classroom as "blank sheets" (Resnick, 1983). Instead, they come with concepts and principles constructed from everyday experience and previous classes. These are very essential for further successful learning. However, some of the mathematical ideas that students have already constructed and use to make sense of the new learning, may be incomplete half-truths. These are 'misunderstandings' or 'misconceptions; in mathematics.

'Misunderstanding' is a problem for two reasons.

- First, they interfere with learning when students use them to interpret new experiences.
- Second, students are emotionally and intellectually attached to their 'misconceptions', because they have actively constructed them.

There are many misunderstandings in elementary mathematics. These have to be identified through assessments on conceptual understandings and clarified for effective new learning of mathematical ideas.

This area should find a place in the curriculum of mathematics education of elementary level. Remedial techniques should be suggested. Students-teachers should be guided to take care of such mathematical misunderstandings of students in elementary classrooms.

It is not surprising to note that student-teachers who come to D.Ed. course also have such misunderstandings. Teacher educators should identify them and follow constructive approaches to provide remedial treatment.

6) Psychology of learning of mathematics

Constructivist learning should be the central focal point in this curriculum area.

Constructivism is recommended by all the recent policy and curriculum documents such as NCF 2005, NCFTE 2009, and curriculum proposed by NCTE and also survey report of status study on D.Ed. curriculum by Karnataka Jnana Aayoga.

Theories and processes of mathematics learning according to Piaget, Bruner, Asubal, Vygotsky, Dienes and Van Hieles should be included. Designing learning environments for mathematics to be based on these theories and processes should be highlighted. **Special reference should be made regarding mathematics learning at "Concrete operation stage" according to Piaget.**

Some of the characteristics are:

- Basic operations like addition, substraction, multiplication and Division are learnt
- Child can learn to solve conservation problems.
- Reversibility is attained.
- Major change is that the intellectual development proceeds from pre-logical though to logical operations and solutions to concrete problems.

Vygotsky's ZPD concept and socio-cultural theory of learning should be specially mentioned related to mathematics learning.

7) Transactional Mode

The catch word in Educational scenario at present is "**Constructivism**". NCF for school Education, 2005 and NCF for teacher education, 2009 have recommended **constructivist** approaches for mathematics learning as well as mathematics teacher education.

This has to be predominantly highlighted for transacting D.Ed. curriculum. D.Ed. classrooms have to inculcate constructivist learning environments for student-teachers to construct mathematics knowledge and pedagogical knowledge of mathematics. This will further enable them to design constructivist learning environments for mathematics in school level classrooms.

Basic assumptions for incorporating constructivist approaches are,

- Knowledge is constructed from experience.
- Learning is a personal interpretation of the world.
- Learning is an active process in which meaning is developed on the basis of experience.
- Conceptual growth comes from the negotiation of meaning, the sharing of multiple perspectives and the changing of our internal representations through Collaborative learning.
- Learning should be situated in realistic settings; testing should be integrated with the task and not a separate activity.

The implications of constructivist approaches are that co-operative learning strategies and collaborative learning should become the mode of transactional processes at both school level and D.Ed. level.

8) Current practices in schools

During the I year, focus should be on the on-going practices of teaching in lower elementary level. 'Nali Kali' practice based on constructive approaches should be mainly discussed. Student-teachers should observe nail-kali lessons, interact with mentors in schools, plan to handle nail kali lessons under the guidance of mentors as well as teacher educators and execute them during block practice.

Note: As "multi-level and multi-grade teaching are covered along with nail-kali lessons, separate multi-grade lessons need not be planned and practiced.

9) ICT in mathematics education

"Technology can greatly aid the process of mathematical exploration, and clever use of such aids can help engage students", this is the recommendation made by NCF 2005. It is applicable to both schools education and teacher education.

In the D.Ed. course Student-teachers should be exposed to both low technology and high technology aids that can be used to teach mathematics. Some

of the crucial aids that student-teachers should be able to use are black boards, charts, graph boards/charts, geo boards, geometry instrument box, calculators, OHP and transparencies, computers etc. They should be well trained to make PowerPoint presentations, use internet sources, and use the computer operating systems and tools such as Ubuntu, Geogebra, Kturtle, etc.

Position paper on "Teaching of Mathematics", NCF (2005) has also suggested that "Innovations in the design and use of such instructional materials must be encouraged so that their use makes school mathematics enjoyable and meaningful".

10) Assessment of mathematics learning

It is a well-known fact that undue importance is given to examination and results in the D. Ed course. This holds good for school education also. A status study on D.Ed course by Karnataka Jnana Ayoga has also reported that "student-teachers and Teacher educators are geared entirely towards examinations". This practice has to be discouraged by completely reforming the assessment and evaluation procedures at D.Ed level. As constructivist approaches have to be the transactional mode in D.Ed. course as well as school education, assessment procedures should be accordingly implemented.

In order to assess the conceptual understanding and competency levels of student-teachers variety of assessment tools such as achievement tests, diagnostic tests, observation records, project work, Seminar, discussions, checklist, student-portfolios, self-assessment, peer-group assessment should be thought of for effective use. Performance-based assessment should be emphasised and a continuous and comprehensive evaluation should be focussed.

This will also enable student-teachers to use similar tools in elementary school classrooms. Hands- on experiences should be provided to student-teachers for constructing classroom-based assessment tools.

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